UNIT III

KINEMATICS OF CAM
INTRODUCTION

CLASSIFICATION OF CAMS AND FOLLOWERS

v A cam is a mechanical element used to drive another element, called the follower, through a specified motion by direct contact.

v Cam-and-follower mechanisms are simple and inexpensive, have few moving parts, and occupy a very small space. Furthermore, follower motions having almost any desired characteristics are not difficult to design.

v For these reasons cam mechanisms are used extensively in modern machinery. The versatility and flexibility in the design of cam systems are among their more attractive features, yet this also leads to a wide variety of shapes and forms and the need for terminology to distinguish them.

Cams are classified according to their basic shapes. Figure 1 illustrates four different types of cams:

1. Plate cam, also called a disk cam or a radial cam
2. Wedge cam
3. Cylindrical cam or barrel cam

Cam systems can also be classified according to the basic shape of the follower. Figure 2 shows plate cams acting with four different types of followers:

1. A knife-edge follower
2. A flat-face follower
3. A roller follower
4. A spherical-face or curved-shoe follower

v Notice that the follower face is usually chosen to have a simple geometric shape and the motion is achieved by proper design of the cant shape to mate with it.

v This is not always the case, and examples of inverse cams, where the output element is machined to a complex shape, can be found.

v Another method of classifying cams is according to the characteristic output motion allowed between the follower and the frame.

v Thus, some cams have reciprocating (translating) followers, as in Figs. 1 a through 1 d and Figs. 2a and 2b, while others have oscillating (rotating) followers, as in Fig. 1c and
Figs. 2c and 2d. Further classification of reciprocating followers distinguishes whether the centerline of the follower stem relative to the center of the cam is offset, as in Fig. 2a, or radial, as in Fig. 2b.

**Figure: 1 Types of cams:**
(a) plate cam; (b) wedge cam; (c) barrel cam; (d) face cam

**Fig: 2 Plate came with**
(a) an offset reciprocating knife edge follower; (b) reciprocating flat face follower; (c) oscillating roller followers; (d) oscillating curved-shoe follower
In all cam systems the designer must ensure that the follower maintains contact with the cam at all times. This can be done by depending on gravity, by the inclusion of a suitable spring, or by a mechanical constraint. In Fig. 1c the follower is constrained by the groove.

Figure 3a shows an example of a constant-breadth cam, where two contact points between the cam and the follower provide the constraint. Mechanical constraint can also be introduced by employing dual or conjugate cams in an arrangement like that illustrated in Fig. 3b. Here each cam has its own roller, but the rollers are mounted on a common follower.

Fig 3: (a) a constant-breadth cam with a reciprocating flat face follower  
(b) conjugate cams with an oscillating roller follower

DISPLACEMENT DIAGRAMS

In spite of the wide variety of cam types used and their differences in form, they also have certain features in common which allow a systematic approach to their design. Usually a cam system is a single-degree-of-freedom device.

It is driven by a known input motion, usually a shaft which rotates at constant speed, and it is intended to produce a certain desired output motion for the follower.

In order to investigate the design of cams in general, we will denote the known input motion by \( \theta (t) \) and the output motion by \( y \). Reviewing Figs. 1 to 3 will demonstrate the definitions of \( y \) and \( IJ \) for various types of cams. These figures also show that \( y \) is a transnational distance for a reciprocating follower but is an angle for an oscillating follower.

During the rotation of the cam through one cycle of input motion, the follower executes a series of events as shown in graphical form in the displacement diagram of Fig. 3-4. In such a diagram the abscissa represents one cycle of the input motion \( \theta \) (one revolution of the cam) and is drawn to any convenient scale.
The ordinate represents the follower travel y and for a reciprocating follower is usually drawn at full scale to help in layout of the cam. On a displacement diagram it is possible to identify a portion of the graph called the rise, where the motion of the follower is away from the cam center.

The maximum rise is called the lift. Periods during which the follower is at rest are referred to as dwells, and the return is the period in which the motion of the follower is toward the cam center.

Many of the essential features of a displacement diagram, such as the total lift or the placement and duration of dwells, are usually dictated by the requirements of the application.

There are, however, many possible follower motions, which can be used for the rise and return, and some are preferable to others depending on the situation.

One of the key steps in the design of a cam is the choice of suitable forms for these motions.

Once the motions have been chosen, that is, once the exact relationship is set between the input $\theta$ and the output y, the displacement diagram can be constructed precisely and is a graphical representation of the functional relationship

$$y = y(\theta)$$

This equation has stored in it the exact nature of the shape of the final cam, the necessary information for its layout and manufacture, and also the important characteristics, which determine the quality of its dynamic performance.

Before looking further at these topics, however, we will display graphical methods of constructing the displacement diagrams for various rise and return motions.

The displacement diagram for uniform motion is a straight line with a constant slope. Thus, for constant input speed, the velocity of the follower is also constant.
This motion is not useful for the full lift because of the corners produced at the boundaries with other sections of the displacement diagram. It is often used, however, between other curve sections, thus eliminating the corners.

The displacement diagram for a modified uniform motion is illustrated in Fig. 5a. The central portion of the diagram, subtended by the cam angle $\beta_2$ and the lift $L_2$ is uniform motion.

The ends, angles $\beta_1$ and $\beta_3$ and corresponding lifts $L_1$ and $L_3$ are shaped to deliver parabolic motion to the follower. Soon we shall learn that this produces constant acceleration. The diagram shows how to match the slopes of the parabolic motion with that of the uniform motion.

With $\beta_1$, $\beta_2$ and $\beta_3$ and the total lift $L$ known, the individual lifts $L_1$, $L_2$ and $L_3$ are found by locating the midpoints of the $\beta_1$, and $\beta_3$ sections and constructing a straight line as shown.

Figure 5b illustrates a graphical construction for a parabola to be fit within a given rectangular boundary defined by $L_1$ and $\beta_1$.

The abscissa and ordinate are first divided into a convenient but equal number of divisions and numbered as shown. The construction of each point of the parabola then follows that shown by dashed lines for point 3.

In the graphical layout of an actual cam, a great many divisions must be employed to obtain good accuracy. At the same time, the drawing is made to a large scale, perhaps 10 times size.

However, for clarity in reading, the figures in this chapter are shown with a minimum number of points to define the curves and illustrate the graphical techniques.

The displacement diagram for simple harmonic motion is shown in Fig.6. The graphical construction makes use of a semicircle having a diameter equal to the rise $L$. 

Figure 5: Parabolic motion displacement diagram: (a) interfaces with uniform motion; (b) graphical construction
The semicircle and abscissa are divided into an equal number of parts and the construction then follows that shown by dashed lines for point 2.

Figure 6: simple harmonic motion displacement diagram; graphical construction

Figure 7: Cycloidal ion displacement diagram; graphical construction

Cycloidal motion obtains its name from the geometric curve called a cycloid. As shown in the left of Fig. 7, a circle of radius $L/2\pi$, where $L$ is the total rise, will make exactly one revolution when rolling along the ordinate from the origin to $y = L$. A point $P$ of the circle, originally located at the origin, traces a cycloid as shown.

If the circle rolls without slip at a constant rate, the graph of the point's vertical position $y$ versus time gives the displacement diagram shown at the right of the figure. We find it much more convenient for graphical purposes to draw the circle only once, using point $B$ as a center.

After dividing the circle and the abscissa into an equal number of parts and numbering them as shown, we project each point of the circle horizontally until it intersects the
ordinate; next, from the ordinate, we project parallel to the diagonal OB to obtain the corresponding point on the displacement diagram.

**GRAPHICAL LAYOUT OF CAM PROFILES**

- Let us now examine the problem of determining the exact shape of a cam surface required to deliver a specified follower motion.

- We assume here that the required motion has been completely determined—graphically, analytically, or numerically—as discussed in later sections.

- Thus a complete displacement diagram can be drawn to scale for the entire cam rotation. The problem now is to layout the proper cam shape to achieve the follower motion represented by this displacement diagram.

- We will illustrate for the case of a plate cam as shown in Fig. 8. Let us first note some additional nomenclature shown on this figure.

- The trace point is a theoretical point of the follower; it corresponds to the point of a fictitious knife-edge follower. It is chosen at the center of a roller follower or on the surface of a flat-face follower.

- The pitch curve is the locus generated by the trace point as the follower moves relative to the cam. For a knife-edge follower the pitch curve and cam surface are identical. For a roller follower they are separated by the radius of the roller.

- The prime circle is the smallest circle that can be drawn with center at the cam rotation axis and tangent to the pitch curve. The radius of this circle is $R_o$.

- The base circle is the smallest circle centered on the cam rotation axis and tangent to the cam surface. For a roller follower it is smaller than the prime circle by the radius of the roller, and for a flat-face follower it is identical with the prime circle.

- In constructing the cam profile, we employ the principle of kinematics inversion, imagining the cam to be stationary and allowing the follower to rotate opposite to the direction of cam rotation.

- As shown in Fig. 8, we divide the prime circle into a number of segments and assign station numbers to the boundaries of these segments.

- Dividing the displacement-diagram abscissa into corresponding segments, we can then transfer distances, by means of dividers, from the displacement diagram directly onto the cam layout to locate the corresponding positions of the trace point.
A smooth curve through these points is the pitch curve. For the case of a roller follower, as in this example, we simply draw the roller in its proper position at each station and then construct the Cam profile as a smooth curve tangent to all these roller positions.

Figure 8: Cam nomenclature

Figure 9: Graphical layout of a plate cam profile with an offset reciprocating roller follower
Figure 9 shows how the method of construction must be modified for an offset roller follower. We begin by constructing an offset circle, using a radius equal to the amount of offset.

After identifying station numbers around the prime circle, the centerline of the follower is constructed for each station making it tangent to the offset circle.

The roller centers for each station are now established by transferring distances from the displacement diagram directly to these follower centerlines, always measuring outward from the prime circle.

An alternative procedure is to identify the points 0', 1', 2', etc., on a single follower centerline and then to rotate them about the cam center to the corresponding follower centerline positions.

In either case the roller circles can be drawn next and a smooth curve tangent to all roller circles is the required cam profile.

**DERIVATIVES OF THE FOLLOWER MOTION**

We have seen that the displacement diagram is plotted with the follower motion $y$ as the ordinate and the cam input motion $\theta$ as the abscissa no matter what the type of the cam or follower. The displacement diagram is therefore a graph representing some mathematical function relating the input and output motions $f$ the cam system. In general terms, this relationship, is

$$Y = y(\theta) \rightarrow (3.1)$$

If we wished to take the trouble, we could plot additional graphs representing derivatives of $y$ with respect to $\theta$. The first derivative we will denote as $y'$:

$$y'(\theta) = \frac{dy}{d\theta} \rightarrow (3.2)$$

It represents the slope of the displacement diagram at each position $\theta$. This derivative, although it may now seem of little practical value, is a measure of "steepness" of the displacement diagram.

We will find in later sections that it is closely related to the mechanical advantage of the cam system and manifests itself in such things as pressure angle. If we consider a wedge cam (Fig.1 b) with a knife-edge follower, the displacement diagram itself is of the same shape as the corresponding cam.
Here we can begin to visualize that difficulties will occur if the cam is too steep, that is, if $y'$ has too high a value.

The second derivative of $y$ with respect to $\theta$ is also significant. It is denoted here as $y''$:

$$y''(\theta) = \frac{d^2y}{d\theta^2} \rightarrow (3.3)$$

Although it is not quite as easy to visualize, this derivative is very closely related to the radius of curvature of the cam at various points along its profile.

Since there is an inverse relationship, as $y''$ becomes very large, the radius of curvature becomes very small; if $y''$ becomes infinite, the cam profile at that position becomes pointed, a highly unsatisfactory condition from the point of view of contact stresses between the cam and follower surfaces.

The next derivative can also be plotted if desired:

$$y'''(\theta) = \frac{d^3y}{d\theta^3} \rightarrow (3.4)$$

Although it is not easy to describe geometrically, this is the rate of the change of $y''$, and we will see below that this derivative should also be controlled when choosing the detailed shape of the displacement diagram.

**Example 3.1** Derive equations to describe the displacement diagram of a cam which rises with parabolic motion from a dwell to another dwell such that the total lift is $L$ and the total cam rotation angle is $\beta$. Plot the displacement diagram and its first three derivatives with respect to cam rotation.

Solution As illustrated in Fig. 5a, two parabolas will be required, meeting at an inflection point taken here at midrange. For the first half of the motion we choose the general equation of a parabola,

$$y = A \theta^2 + B \theta + C \ldots \ldots \ldots \ldots (a)$$

which has derivatives

$$y' = 2A \theta + B \quad (b)$$

$$y'' = 2A \quad (c)$$

$$y''' = 0 \quad (d)$$
To match the position and slope with those of the preceding dwell properly, at \( \theta = 0 \) we have \( y(0) = y'(0) = 0 \). Thus, Eqs. (a) and (b) show that \( B = C = 0 \). Looking next at the inflection point, at \( \theta = \beta/2 \) we want \( y = L/2 \); Eq. (a) yields

\[
A = \frac{2L}{\beta^2}
\]

Thus, for the first half of the parabolic motion, the equations are

\[
y = 2L \left( \frac{\theta}{\beta} \right)^2
\]

\[
y' = \frac{4L}{\beta} \frac{\theta}{\beta}
\]

\[
y'' = \frac{4L}{\beta^2}
\]

\[
y''' = 0
\]

The maximum slope occurs at the inflection point, where \( \theta = \beta/2 \). Its value is

\[
y'_{\text{max}} = \frac{2L}{\beta}
\]

For the second half of the motion we return to the general equations (a) through (d) for a parabola. Substituting the conditions that at \( \theta = \beta \), \( y = L \), and \( y' = 0 \), we have

\[
L = AB^2 + B\beta + c
\]

\[
0 = 2AB + B
\]

Since the slope must match that of the first parabola at \( \theta = \beta/2 \), we have, from Eqs. (5-9) and (b),

Solving Eqs. (e) through (g) simultaneously gives

\[
A = -\frac{2L}{\beta^2} \quad B = \frac{4L}{\beta} \quad C = -L
\]

When these constants are substituted into the general forms, we obtain the
From the general equation of the displacement diagram,
\[ y = y(\theta) \quad \theta = \theta(t) \]
We can therefore differentiate to find the time derivatives of the follower motion. The velocity of the follower, for example, is given by
\[ \dot{y} = \frac{dy}{dt} = \frac{dy}{d\theta} \frac{d\theta}{dt} \]
\[ \dot{y} = y' \omega \] (5-14)
Similarly, the acceleration and jerk of the follower are given by
\[ \ddot{y} = \frac{d^2y}{dt^2} = y'' \omega^2 + y'\alpha \] (5-15)
and
\[ \dddot{y} = \frac{d^3y}{dt^3} = y''' \omega^3 + 3y'' \omega \alpha + y'\dot{\alpha} \] (5-16)
When the camshaft speed is constant, these reduce to
\[ \dot{y} = y' \omega \quad \ddot{y} = y'' \omega^2 \quad \dddot{y} = y''' \omega^3 \] (5-17)
For this reason, it has become somewhat common to refer to graphs of the kinematics derivatives \( y', y'', \) and \( y''' \), such as those shown in Fig. 12, as the "velocity," "acceleration," and "jerk" curves for a given motion.
They would be appropriate names for a constant-speed cam only, and then only when scaled by \( \omega, \omega^2, \) and \( \omega^3 \), respectively. However, it is helpful to use these names for the derivatives when considering the physical implications of a certain choice of displacement diagram. For the parabolic motion of Fig. 12, for example, the "velocity" of the follower rises linearly to a maximum and then decreases to zero.
The "acceleration" of the follower is zero during the initial dwell and changes abruptly to a constant positive value upon beginning the rise. There are two more abrupt changes in "acceleration" of the follower, at the midpoint and end of the rise. At each of the abrupt changes of "acceleration," the "jerk" of the follower becomes infinite.

**HIGH-SPEED CAMS**
Continuing with our discussion of parabolic motion, let us consider briefly the implications of the "acceleration" curve of Fig. 10 on the dynamic performance of the cam system.
Any real follower must, of course, have some mass and, when multiplied by acceleration, will exert an inertia force. Therefore, the "acceleration" curve of Fig. 10 can also be
thought of as indicating the inertia force of the follower, which, in turn, must be felt at the follower bearings and at the contact point with the cam surface.

An "acceleration" curve with abrupt changes, such as parabolic motion, will exert abruptly changing contact stresses at the bearings and on the cam surface and lead to noise, surface wear, and eventual failure.

Figure 10: (a) Circle-arc cam (b) Tangent cam

Thus it is very important in choosing a displacement diagram to ensure that the first and second derivatives, the "velocity" and "acceleration" curves, are continuous, that is, that they contain no step changes.

Sometimes in low-speed applications compromises are made with the velocity and acceleration relationships. It is sometimes simpler to employ a reverse procedure and design the cam shapes first, obtaining the displacement diagram as a second step.

Such cams are often composed of some combination of curves such as straight lines and circular arcs which are readily produced by machine tools. Two examples are the circle-arc cam and the tangent cam of Fig.10. The design approach is by iteration.

A trial cam is designed and its kinematics characteristics computed. The process is then repeated until a cam with the desired characteristics is obtained. Points A, B, C, and D of the circle-arc and tangent cams are points of tangency or blending points. It is worth noting, as with the parabolic-motion example above, that the acceleration changes abruptly at each of the blending points because of the instantaneous change in radius of curvature.
Although cams with discontinuous acceleration characteristics are sometimes found in low-speed applications, such cams are certain to exhibit major problems as the speed is raised.

For any high-speed cam application, it is extremely important that not only the displacement and "velocity" curves but also the "acceleration" curve be made continuous for the entire motion cycle. No discontinuities should be allowed at the boundaries of different sections of the cam.

How high a speed one must have before considering the application to require high-speed design techniques cannot be given a simple answer.

This depends not only on the mass of the follower but also on the stiffness of the return spring, the materials used, the flexibility of the follower, and many other factors. Still, with the methods presented below, it is not difficult to achieve continuous derivative displacement diagrams. Therefore it is recommended that this be done as standard practice.

Parabolic-motion cams are no easier to manufacture than Cycloidal motion cams, for example, and there is no good reason for their use. The circle-arc and tangent cams are easier to produce, but with modern machining methods cutting of more complex cam shapes is not expensive.

STANDARD CAM MOTIONS

Details of the equations for parabolic motion and its derivatives have already been seen in the previous sections.

The purpose of this section is to present equations for a number of standard types of displacement curves which can be used to address most high-speed cam-motion requirements.

The displacement diagram and its derivatives for simple harmonic rise motion are shown in Fig. 11. The equations are

\[ y = \frac{L}{2} \left( 1 - \cos \frac{\pi \theta}{\beta} \right) \]

\[ y' = \frac{\pi L}{2 \beta} \sin \frac{\pi \theta}{\beta} \]

\[ y'' = \frac{\pi^2 L}{2 \beta^2} \cos \frac{\pi \theta}{\beta} \]

\[ y''' = -\frac{\pi^3 L}{2 \beta^3} \sin \frac{\pi \theta}{\beta} \]

Unlike parabolic motion, simple harmonic motion shows no discontinuity at the inflection point.